# 1. Introduction

~~In physics and mathematics, Green'~~Green's theorem ~~gives~~indicates the relationship between a line integral around a simple closed curve C and a double integral over the ~~plane~~plane region D bounded by C. This theorem is an application of the fundamental ~~Theorem~~theorem of calculus to integrat~~ing~~e a certain combination~~s~~ of derivatives over a plane. It can be ~~proven~~ easily proven for rectangular and triangular regions. As ~~B~~both sides of ~~it's~~its equality are finitely additive and almost all planar regions can be divided into triangles and rectangles, ~~so that~~ the result holds for any planar~~-~~ region ~~practically all of, which can be divided in to triangles and rectangles~~. This proves ~~—~~the theorem for reasonably shaped regions. ~~It's~~ Its generalization to ~~the~~ non~~-~~planar surfaces (~~— proved~~ directly proved ~~from it by~~ using the finite additivity of both sides~~-~~) is the Stokes' ~~Theorem~~ theorem described below.

Comment [A1]: If the first letter of a word has a vowel sound, "an" should be used, If the first letter has a consonant sound, "a" should be used.

Comment [A2]: Providing concise and clear sentences often aids clarity and enhances readability.

## 1.1 ~~Greens~~Green's Theorem

~~Its~~ The formal statement of Green's theorem is as follows~~— : —~~: Let S be a sufficiently nice region in the plane, and let $\delta$ S be its boundary~~;then~~. Then, we have~~;~~

$$\iint_S \left(\frac{\partial v_2}{\partial x} - \frac{\partial v_1}{\partial y}\right) dx\,dy = \oint_{\delta S} (v_1 dx + v_2 dy)$$

,

where the boundary~~,~~ $\delta$ S is traversed counterclock~~-~~wise on ~~it's~~its outside cycle~~,~~ (and clockwise on any internal cycles as you can verify using zippers~~.)~~).

Comment [A3]: Sentences should begin with the noun, following by its pronoun in subsequent sentences.

~~Meaning of this t~~Theorem interpretation: ~~Green's~~Green's theorem is a form that the fundamental theorem of calculus ~~take~~takes in the context of integrals over planar regions.

Comment [A4]: In academic writing, information is presented with accuracy and conciseness. Formal language is a hallmark of academic English. One way to ensure conciseness in expression is converting phrasal verbs to formal words.

**For a rectangle:** By the ordinary fundamental theorem of calculus, we have~~;~~

$$\int_{x=a}^{x=b}\int_{y=c}^{y=d} \frac{\partial v_2(x,y)}{\partial x} - \frac{\partial v_1(x,y)}{\partial y}\, dy\, dx = \int_{y=c}^{y=d} (v_2(b,y) - v_2(a,y))\,dy - \int_{x=a}^{x=b} (v_1(x,d) - v_1(x,c))\,dx$$ .

**For a right triangle:** ~~f~~For convenience, we choose a triangle bounded by line x = 0, y = 0, and $\frac{x}{a} + \frac{y}{b} = 1$.

We similarly get~~:~~

$$\int_{y=0}^{y=b}\int_{x=0}^{x=a-ya/b} \frac{\partial v_2(x,y)}{\partial x}\, dx\, dy - \int_{x=0}^{x=a}\int_{y=0}^{y=b-xb/a} \frac{\partial v_1(x,y)}{\partial y}\, dy\, dx$$
$$= \int_{y=0}^{y=b} (v_2(a - ay/b, y) - v_2(0,y))\,dy - \int_{x=0}^{x=a} (v_1(x, b - bx/a) - v_1(x,0))\,dx$$ .

Comment [A5]: To use the colon correctly, you must make sure that sentence that comes before the colon is a complete, grammatical sentence.

Rearrangement of the right hand side gives the ~~Theorem~~theorem for rectangles and right triangles.

It means that~~,~~ for R, a rectangle or right triangle in the x-y plane~~,~~ (for which d**S** = dS**k**), we have

$$\iint_R \overline{\nabla} \times \overline{v} \bullet d\overline{S} = \oint_{\delta R} \overline{v} \bullet d\overline{l}$$ .

Both sides of this equation ~~is finite~~are finitely additive~~:~~, ~~that is~~i.e., if we take two disjoint regions~~,~~ and evaluate either one over both, ~~you~~we get the sum of their values on the two regions ~~separate .~~separately. This is true even if the regions share a common boundary~~,~~ because the line integrals will cancel out over the common boundary ~~which~~ that ceases to be a boundary.

Comment [A6]: In American English, "that" is used to introduce a restrictive clause and "which" a nonrestrictive clause.

The result follows from additivity for any region that can be broken up into rectangles and triangles, which accounts for most regions we will encounter.